The Method of Lines for the Rigorous Full Wave Analysis of Rectangular Bends of Multiple-Line-Systems

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Abstract

A new full-wave analysis for the consistent investigation of the electrical characteristics of rectangular bends of coupled lines using the Method of Lines is presented. The scattering matrices and the equivalent circuits are determined. As an application the analysis is used in the investigation of the electrical properties of rectangular spiral inductors.

1 Introduction

In the microwave technology many passive components with complex structures of planar striplines are used. Analyzing them in "one piece" is often not sensible and necessary. The field distribution around the striplines as well as the shape of such components often allow a partition of the structure in suitable, smaller partial circuits, which in common demand only easier analyses. Nevertheless the analyses have to consider all electrical properties as they appear in the original structure.

In components, which are constructed of discontinously guided lines often rectangular bends of coupled lines are required. One-line bends appear e.g. in meanderlines, multiple-line bends e.g. in couplers, filters, inductance coils and line systems of digital busses.

The properties of these components are in most cases essentially determined by the influence of the bends. Hence accurate analyses of the electrical characteristics of the bends are important for the investigation of the complete structures.

In many different works of other authors, e.g. Easter and Gopinath [1], Thomson and Gopinath [2], Neale and Gopinath [3], the electrical properties of one-line-bends are treated.

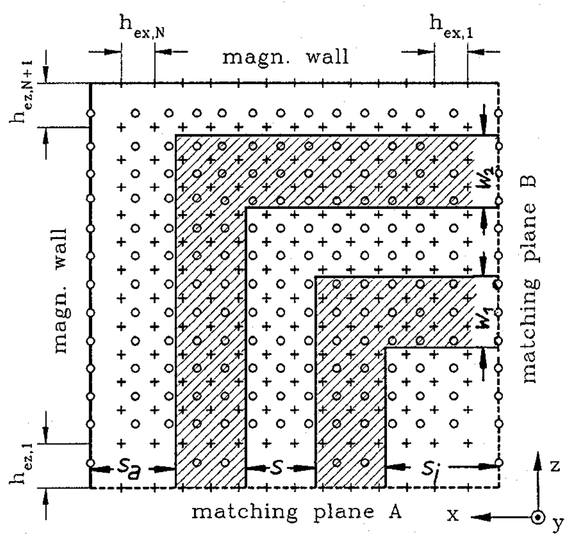

Figure 1: Example of a corner element with a rectangular bend of a two line system and its discretization.
$+ \Rightarrow E_z$; $\circ \Rightarrow H_z$;

In this contribution a new rigorous full wave analysis of rectangular bends of coupled multiple-line systems is presented, which investigates the fields in a spatial domain around the bends in "one piece". The authors do not know about other analyses, which treat bends of coupled lines in a similar way.

The base of the analysis originates from the source method of Worm [4]. This method is extended in some essential parts.

The analyzed model, in the following named a corner element, is shown in fig. 1 for the example of M=2 lines. The corner element is symmetrically to the plane x=z and contains the infinitely thin lines between two dielectric layers. The ground and ceiling are metallized, two lateral walls are chosen as magnetic ideal conducting planes. At the remaining two lateral walls, the matching planes A and B, straight multiple-line systems are connected. At these matching planes the fields of the twodimensional cross-section analyses [5] of the straight multiple-line systems are used as inhomogenous boundary conditions.

The field excitations at both matching planes are distinguished into two fundamental cases, the symmetric and the antisymmetric. In the first named case of excitation the E-fields are given symmetrically and in the last named case antisymmetrically at both matching planes. Corresponding the H-fields are antisymmetrically in the first named case and symmetrically in the last named case.

It is assumed, that the distances from the inner bend of the line system to the matching planes are so large, that all excited nonpropagation modes have disappeared at the matching planes. The investigated frequency-range is limited by the cutoff-frequency of the first higher mode.

1 Analysis

The analysis with the Method of Lines consist of the following six essential steps.

- The non-equidistant discretization is applied onto the fields and the field- and wave-equations. The derivations of the field components parallel to the strip plane are substituted by the differences between the field components on the discretization lines.
- With suitable transformations the field elements in the wave-equations are decoupled, leading to systems of ordinary, inhomogenous differential equations, which depend only from the axis perpendicular to the strip plane (y-direction in fig. 1). The inhomogenous differential equations are solved analytically and result in accurate relations for the fields along the discretization lines.
- In the transform domain relations between the tangential fields are derived.
- To prepare the field matching, the transformed fields within the corner element, but not the transformed excitation fields, are transformed back to the spatial domain.
- For the field matching the tangential fields outside the strips in the common interface of both layers must be set equal. On the strips the tangential electric fields are zero. After reduction of the resulting systems of equation the current densities on the strips are determined in some further mathematical steps. Then the field distribution within the corner element is derived.
- From this field distribution and under consideration of the excitation fields at the matching planes the phasors of the forward and backward travelling waves of all propagation modes are calculated. To obtain the scattering matrix M calculations must be carried out with different excitation fields. The resulting phasors of the forward and backward travelling waves are combined and from them the scattering matrix is determined.

2 Results

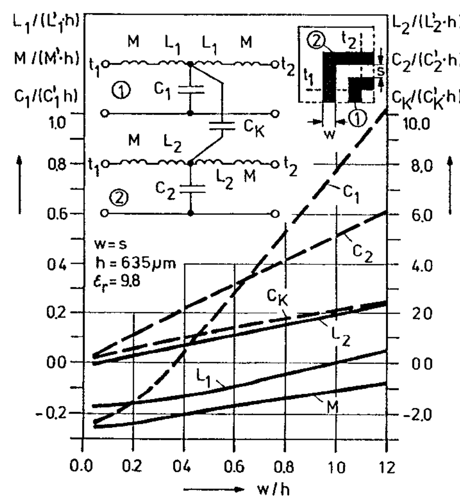

Figure 2: Values of the elements of the equivalent circuit of a 2-line bend, related to the values of the elements per unit length and the substrat thickness h, depending on the ratio w/h of stripwidth to substrat thickness.

The results of the numerical calculations of the electrical properties of 1-line bends are compared with those

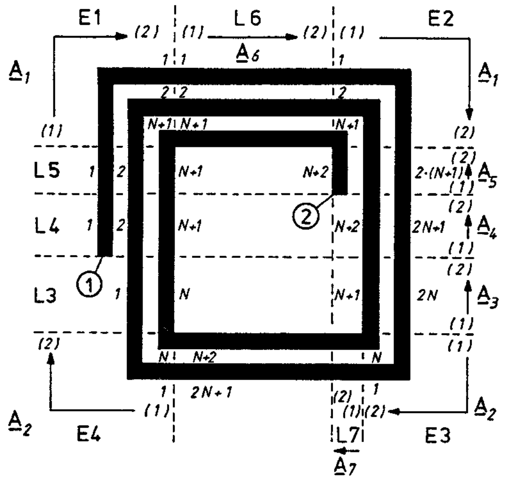

Figure 3: Model of an spiral inductor with partitioning in straight and bent multiple line sections.
E1 - E4 : bent multiple-line sections
L3 - L7 : straight multiple-line sections
$\boldsymbol{A}_1 \cdots \boldsymbol{A}_7$: ABCD-matrices of the line sections with input (1) and output (2) declaration.

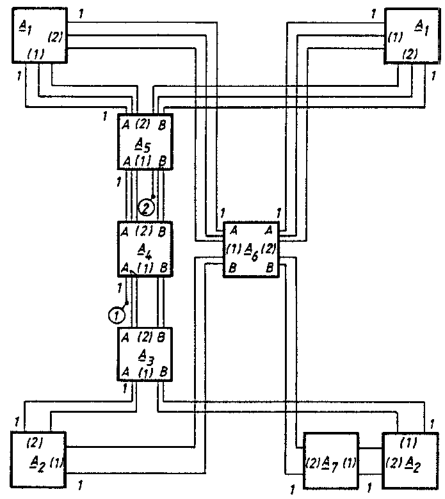

Figure 4: Connections between the ABCD- matrices $\boldsymbol{A}_1 \cdots \boldsymbol{A}_7$ for the spiral inductor in fig. 3.

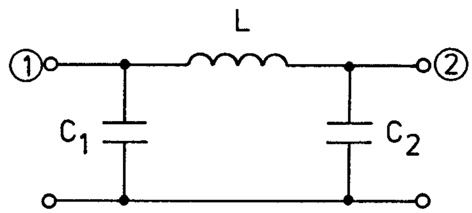

Figure 5: Equivalent circuit of a spiral inductor

of other authors. These comparisons and new results, concerning the electrical characteristics of 2- and 3-line bends are presented and described. For 2-line bends equivalent circuits are calculated. E.g. in figure 2 the results of the investigation of the effect of varying ratios w/h of stripwidth to substrat thickness onto the values of the elements are shown.

In further analyses the electrical properties of rectangular spiral inductors are investigated.

Two models are analyzed which differ in the treatment of the mutual inductances between the lines of different spiral sides. In both models the analysis of the electrical characteristics of the multiple-line bends is considered.

The first model (A) consist just of the series connected multiple-line sections, ignoring the mutual inductances between the lines of different spiral sides. The second model (B), shown in fig. 3, is composed of the straight multiple-line sections L3-L7, for which all self- and mutual inductances are taken into account and of the bent multiple-line sections E1-E4. For the interconnection the ABCD-matrices of all sections are determined. By some simple extensions of the used analyses the ABCD-matrices of the different multiple-line sections are easily derived. Due to the same ABCD-matrix for E1, E2 and E3, E4, respectively, in the example presented, namely a 2.5-turn spiral inductor, it is sufficient to calculate the ABCD-matrices $\underline{\boldsymbol{A}}_1$ - $\underline{\boldsymbol{A}}_7$. The interconnection of the ABCD-matrices for the example of fig. 3 is shown in fig. 4. From this arrangement the ABCD-matrix of the spiral inductor, related to its connections (1,2 in fig. 4) is determined.

For the spiral inductors the simple equivalent circuit (fig. 5) is used and its elements are determined from the ABCD-matrix of the complete model.

For the accurate analyses of the spiral inductors the results of the analysis of the coupled bends have been applied. Different spiral inductors using the models (A) and (B) have been investigated. The results of these investigations are compared with those of other authors, e.g. Cahana [6], Djordjevic et. al. [7] and Shepherd [8] respectively.

Cahana's result for a 1.75-turn spiral inductor agree well with the result of the comparing model A, but deviate much to the result using the more accurate model B. As the substrate thickness of this investigated spiral inductor is greater than the comparatively small diameter, strong mutual inductances between the lines of different spiral sides appear. These are considered in model B,

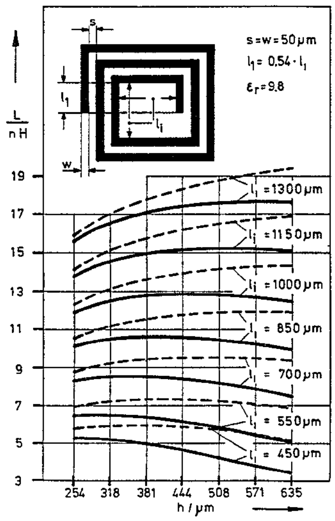

Figure 6: Values of the series inductance of a 2.5-turn spiral inductor versus the substrate thickness h for different inner side lengths l_i.
(model A: ‒ ‒ ‒ ‒ , model B: ———)

but are neglected in model A and Cahana's model.

On the other hand the results of Djordjevic et. al. for a one-turn and two-turn spiral inductor as well as the result for a 3.25-turn spiral inductor of Shepherd agree fairly well with those results calculated in this analysis.

As a new result, for a 2.5-loop spiral inductor the values of the series inductance for different inner side lengths of the spiral inductor versus varying substrate thicknesses are shown in fig. 6. The deviations between the results of both models show clearly the influence of the mutual inductances between lines of different spiral sides. These mutual inductances cannot be neglected in the analyses of most spiral inductors used in MMIC's owing to the relative great ratios of h/l_i. Only in the analyses of spiral inductors with $h/l_i < 0.2$ it is possible to neglect the mutual inductances. In this case both models yield values of inductivity which are close together.

Analyses of spiral inductors, in which also the effects of the bends are neglected and where the bends are substituted by short line segments yield values of inductivity much too large.

Hence for spiral inductors used in MMIC's accurate analyses have to consider the effects of the bends and the mutual inductances between lines of different spiral sides.

3 Conclusion

Accurate procedures, based on the Method of Lines, are presented for the investigations of bends of coupled lines and of rectangular spiral inductors. The results are compared with those of other authors and new results are presented. For spiral inductors the accuracy of the best of two models used is confirmed by the results of other authors, who also investigated spiral inductors with a comprehensive and accurate analysis.

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